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ABSTRACT

The role of errors in education must be re-examined to allow teachers to exploit fully the educational potential of the error. Student errors have been used as tools to study the learning process and to plan teaching materials. A new metaphor for error making, that of "getting lost," can help uncover some limitations in the current educational use of errors. Developing this metaphor in a generative spirit shows the importance of the context in which the error occurs. Under some conditions "getting lost" can be enjoyable. Errors can both motivate and guide students in intellectual explorations. Tables summarize possible reactions to "getting lost." General strategies undertaken in response to an error or "getting lost" include: (1) asking someone for help; (2) making use of tools and resources; (3) reflecting on the situation; and (4) exploring. (SLD)

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TOWARDS A RECONCEPTUALIZATION
OF THE ROLE OF ERRORS IN EDUCATION:
THE NEED FOR NEW METAPHORS

by

Raffaella Borasi

Paper presented at the Annual Meeting of the

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I. Introduction

In recent years, we have had sophisticated analyses of the notion of problem solving from philosophical, psychological, mathematical and educational communities. As a result, the notion itself of what constitutes a problem and the roles of different types of problems in the learning of a discipline have been examined and debated at length, with great advantages for instruction. Though error-making is a critical component in the activity of problem solving, the concept of "error" has not yet received a similar analysis. I suggest, on the contrary, that the notion of "error" may require such a re-examination, in order to allow teachers to fully exploit its educational potential in formal and informal instruction. Despite the wisdom of the motto "You learn from your mistakes," students rarely perceive errors as anything but a negative event to be avoided whenever possible -- sometimes even at the cost of not trying at all!

A more positive role for errors in education has, however, been recognized in recent years in the areas of mathematics, second language, reading and writing instruction. Student errors have been employed by researchers and teachers as a valuable tool to diagnose learning

difficulties, to study the learning process, and to plan curriculum and teaching material accordingly. Though this approach has certainly proved valuable for research and practice in learning and teaching, I have suggested that it has as yet only partially exploited the educational potential of errors [Borasi, 1986; 1987a]. Even a superficial look at the history of science and at the activities of scientists as well as artists, shows that errors can play an even more important and essential role in the creation of new results and in the attainment of knowledge [Kuhn, 1970; Lakatos, 1976]. How could errors be employed in instruction so that students could fully benefit from them? Answering this question may require a quite radical reconceptualization of the nature and role of errors in education. In this paper I will attempt to provide a contribution in this direction by suggesting a new metaphor for error-making -- "getting lost" in a city -- and by developing its implications for instruction.

It has in fact been recognized that the metaphors we carry with us may considerably affect the way we perceive a phenomenon. Thus it may be expected that the existing metaphors about errors may have informed the way the problem has been defined as well as the kind of solutions which have been pursued so far.

Two metaphors have been commonly employed in the education literature regarding errors. The first and most popular one, revealed by the use of terminology such as diagnosis and remediation, or clinical interview, is clearly borrowed from the medical field. Errors are seen here as the symptoms of a disease (the latter being the student's misconception or learning difficulty that caused the error). More

recently, we have also encountered errors being referred to as "bugs" (especially in the cognitive science and mathematics education literature). This time the metaphor has been borrowed from the field of computer science. Errors are here equated to inappropriate instructions in a computer program which do not allow us to reach the desired outcome.

Both metaphors have greatly contributed to the more positive and constructive approach to errors in education which we have mentioned before. For instance, they informed teachers of the inefficiency of attempting to eliminate student errors simply by reexplaining a topic or by assigning more practice, and pointed out the value of errors as sources of information about students' real problems in learning. At the same time, it is also important to recognize that these metaphors have embedded in them assumptions worth questioning or even challenging. For example, negative connotations are certainly connected with both diseases and bugs in a program. These two metaphors thus implicitly convey the message that errors are something which we need to eliminate and wish to avoid whenever possible. The medical metaphor in particular also brings along with it the dangerous premise that you need an expert -- the teacher or the researcher -- in order to be able to make use of errors, leaving the student quite passive and helpless in the process.

The consideration of "getting lost" as an alternative metaphor for error-making will help us uncover and overcome some limitations in the current educational uses of errors arising from the assumption implicit in the medical or the computer metaphor. More importantly, the development of this metaphor in a "generative" spirit will provide a

unique tool to inquire into the phenomenon of error-making from new perspectives, and to suggest new approaches to student errors in instruction.

The paper will develop as follows. The next section will discuss in further detail the specific use of metaphors which will be employed in the rest of the paper. The in-depth analysis of the situation of "getting lost" and the creation of explicit analogies with error-making in an educational context will then be developed. The paper will conclude with some considerations about the implications for instruction and teacher education that such analysis suggested.

II. On using "generative metaphors" as a method of inquiry

The fundamental role that an analysis of metaphors can play for the understanding of complex concepts and issues has long been recognized. Metaphors have always been employed as a device for explaining and communicating in narrative, poetry and teaching.

However, more recently an analysis of the nature and role of metaphors from non-literary perspectives has also been undertaken, and has revealed the power of metaphors as tools for inquiry and thought in a variety of other fields as well [cfr. Ortony, 1979]. For example, Kuhn [1979] and Boyd [1979] argue that scientists have used metaphors consistently in the creation of scientific models. An implicit use of metaphors in the way in which we "talk" about topics and issues in common language has also been uncovered by philosophers and linguists (see for example Lakoff and Johnson, 1980). Philosophers such as Scheffler have explicitly employed an analysis of the metaphors as an integral part of their literary analysis of fundamental educational concepts [Scheffler, 1960].

In this paper, "error-making as getting lost" will be employed as a "generative metaphor." Since Schon has beautifully described and illustrated this use of metaphors as a means for better understanding a phenomenon and creating new meaning [Schon, 1960 and 1979], in what follows I will make extensive use of his argument and examples, rather than attempting to create new ones.

A metaphor is created whenever we recognize an essential similarity between two things which had been perceived as distinct up to that

point. This first global and intuitive recognition of similarity can become "generative" when, by analyzing and developing such analogy, we are brought to see the original phenomenon in a new light. Schon reports of a remarkable instance of a "generative" metaphor which occurred when a group of researchers was studying paintbrushes made of synthetic bristles. When one of the researchers observed that "a paintbrush is a kind of pump", the group was brought to reexamine the situation from new perspectives and generated totally unexpected solutions. For example, the functioning of the pump suggested to study the curve formed by natural and synthetic bristles and its effect on the way paint could be sprayed evenly; as a result, the scientists were able to suggest a composition and shape for the synthetic bristles which would produce results considerably better than those obtained up to that point.

Schon's analysis of how the metaphor actually helped the scientists redefine and consequently solve their problem is very illuminating:

[Saying that] "a paint-brush is a kind of pump" ... it is as though he was posing a kind of riddle ("How is a paint-brush a pump?") which, once entertained, led him and the other researchers to notice new features of the brush and of the painting process. The constellation of notions familiarly associated with pumping ... [was] project[ed] onto the painting situation, transforming their perception. [Schon, 1979, p.258]

What makes the process one of metaphor making, rather than simply of redescribing, is that the new putative description already belongs to what is initially perceived as a different, albeit familiar thing; hence, everything one knows about pumping has the potential of being brought into play in this redescription of painting. [Schon, 1979, p.259]

As Schon remarks, not all metaphors are necessarily "generative". To exploit this potential of metaphors it is necessary that the global

similarity initially recognized in the act of creating the metaphor itself be then used to see the original phenomenon in a new light, and thus "generate new perceptions, explanations and inventions". This requires a conscious attempt to explicitly identify those elements that make the two things similar, and to apply one's knowledge of one phenomenon in the effort to understand the other:

The making of generative metaphors involves a developmental process. It has a life cycle. In the earlier stages of the life cycle, one notices or feels that A and B are similar, without being able to say similar to respect to what. Later on, one may come to be able to describe relations of elements present in a restructured perception of both A and B which account for the preanalytic detection of similarity between A and B. Later still, one may construct a general model for which a redescribed A and a redescribed B can be identified as instances. To read the later model back onto the beginning of the process would be to engage in a kind of historical revisionism. [Schon, 1979, p.269]

In the following section, the metaphor of "error-making as getting lost" will be developed along these lines. This will be achieved through an in-depth analysis of various aspects of the situation of "getting lost," followed by the drawing of explicit parallels with error-making in an educational context. In most cases, a general model that captures critical elements common to the two phenomena will also be presented. As a result of this analysis, I hope that we will all come to "new realizations and inventions" about how errors can be approached constructively in instruction.

Ideally, to best use the power of metaphors as a tool for thinking, I believe that the audience should be actively engaged in developing the metaphor, rather than passively receiving a report of the same activity conducted by somebody else. I thus encourage the readers to think about

their own experiences when getting lost and to draw their own implications for error-making in an instructional context, before proceeding further in the paper. Nevertheless, I have decided to also report in detail the results of my own analysis of this metaphor, at the risk of being tedious at times. Besides offering the consideration of some elements that the reader may have overlooked as well as a different set of examples, I believe in fact that a thorough and organized analysis of the metaphor may provide an essential framework, which can in turn help to place, interpret, develop and even stimulate further contributions.

III - Developing the metaphor of "error-making as getting lost"

Though I have not encountered so far an explicit recognition of the analogy between "getting lost in a city" and "making an error in an education context", this metaphor is already implicitly embedded in our language. Think for example of expressions such as "I don't know where I went wrong", "At some point I got lost in the teacher's explanation and I could not follow it any more", "I must have taken a wrong turn somewhere, but I realized it only when I was hopelessly lost".

For several reasons, this metaphor immediately appealed to me, and suggested the value of using it in a generative spirit as a tool to rethink our approach to errors in instructions. First of all, "getting lost" is a situation which we have all often experienced and thus we can easily relate to. Secondly, the person getting lost is the one in charge of the analysis of the "error" and of the decision of what to do about it. Thirdly, this metaphor points to the important role played by feelings and emotions associated with error-making.

Yet other elements, which I think make the situation of "getting lost" especially valuable as a metaphor for error-making in an educational context, came to my realization only later, when examining some examples in-depth. Thinking back at my various experiences getting lost in a city, and of my reactions to it, I realized with surprise that I could bring back to mind radically different situations. I remember getting lost in Washington while getting back to my friends' place by car late at night; the overwhelming feeling was one of panic and acute

discomfort, and I would have avoided the experience gladly even though it resulted in my learning about the unusual division of that city in four quadrants. Similarly, I know that I can get very nervous (and consequently less rational and efficient!) if I happen to lose my way when attempting to make an important appointment. However, other experiences of getting lost do not necessarily bring along such discomforting memories. When moving to a new city, getting lost at the beginning is accepted as a fact of life, and - as long as it is daylight, I am not in a hurry and I have a map with me - I can even enjoy the discovery of new parts of the city and the gradual recognition of some landmarks in the process of finding my way back. Getting lost has even occasionally turned into an occasion for adventure when on vacation visiting new places - such episodes in which curious encounters and discoveries happen as a result of getting lost are part of almost any travel to a foreign country.

All these examples point to the importance of the context in which getting lost or making an error may occur, and to the necessity of taking such context into consideration when evaluating alternative strategies to deal constructively with the situation. They also suggest that errors may not necessarily represent a negative event to be avoided, since sometimes an error could even provide the opportunity for valuable experiences and discoveries which one would have missed otherwise.

These considerations immediately challenge the traditional definition of the problem constituted by student errors in instruction -- i.e., "how to eliminate the error and avoid its occurrence in the

future" -- as too narrow and reductive. They also provide a structure for our analysis of the complex phenomenon of "getting lost in a city", by suggesting the following sets of questions:

- (A) What are alternative ways in which the event of "getting lost" can be perceived and interpreted? What circumstances can determine, or at least influence, such decision? What implications would such a decision have on the kind of solutions or outcomes we would strive for?
- (B) What can we do once we realize that we are lost? What are possible constructive and non-constructive strategies for action among which one could choose? What outcomes are they likely to produce? What variables can determine, or at least influence, our decision for one course of action versus another?
- (C) What is the nature of and the role played by the emotional reactions which can accompany the realization of being lost? In particular, what is their relation with the way the event is perceived, with the action we decide to follow, and with other aspects of the context?

With the goal of providing suggestions that will help students make a better use of their own errors in school, throughout this analysis I will also question what we could do, as educators, to provide a learning context which is more compatible with a positive view and use of errors.

III.A) Alternative Scenarios and Expectations

To address the set of questions related to the interpretation of the event of getting lost/error in different situations, I have first of all identified three alternative scenarios. Each scenario will be

described and analyzed focusing on our overall goals in this situation, how these can determine our definition of the problem caused by having gone astray, and what kind of outcomes would be considered desirable. Then, the conditions which determine each scenario will be identified and analyzed. A general scheme that summarizes the relations thus uncovered will conclude this sub section.

Scenario 1: Focus on reaching a specific objective

Needing to reach a specific destination as quickly as possible is a situation which each of us has often experienced -- suppose for instance, that we have to give a lecture, or to make an important appointment. In these circumstances, getting lost will constitute a nuisance and a problem, and our efforts will focus on finding a way to reach the original destination without delay. It is quite conceivable that in this scenario we will be quite reluctant to be sidetracked with any extraneous information, we will not appreciate attempts to make us reach the solution "on our own", nor will we be particularly prone to becoming interested in the place we have involuntarily reached!

A test probably represents the most extreme example of the analogue of this scenario in education: all that matters is getting those answers correct within the allotted time! However, this scenario is operating in other aspects of formal instruction as well, whenever students perceive that their learning will be measured essentially in terms of a product -- how many facts or skills they have mastered, and what exercises or problems they can answer correctly. In all these circumstances, students' and teachers' efforts will be directed to producing the desired product, by eliminating the error or even by

disregarding it and starting afresh if at all appropriate. As the analogy with the corresponding situation when getting lost suggests, there is very little incentive to do something more constructive with the error and to learn from it within this scenario.

Scenario 2: A concern for both product and process

An alternative scenario is represented by going home from work, when we have just moved to a new city. Since this is a path we will cover many times in the future, perhaps with variations as required by picking up a friend or running an errand along the way, our goal will not be limited to "getting there as quickly as possible". Rather, we are probably interested in trying out alternative routes, learn to recognize landmarks and more generally getting to know our way around. In other words, even if we have to reach a specific destination eventually, we have other long-term agendas operating as well.

Compared with the previous scenario, getting lost is certainly less disturbing here, though probably still unwelcomed, and we may also be more ready to appreciate some of its potential benefits. Our definition of the problem in this case will probably include, besides finding a way to reach our original destination from where we are, also gaining an understanding of why we went astray so as to minimize the chances of getting lost again. In addition to getting to our destination eventually, we can also hope to discover an alternative route, gain an increased familiarity with our map of the city and perhaps even a better understanding of how the city is laid out.

A similar perspective seems to be shared in education by those who appreciate the value of learning general procedures beyond getting the

right answers. Acquiring a body of knowledge and solving problems correctly is still considered important, but there is a new emphasis on the "process" as well. Learning from errors acquires some positive role in this context. Though one might prefer not to make errors in the first place, it is also recognized that one can learn from one's errors to minimize failure in the future. So, for example, a student can be encouraged to pay closer attention to the errors made while performing a set of subtractions, so that he can recognize patterns which may in turn allow him to identify the causes of his difficulties and remediate them. Besides correcting his own errors in this set and producing the correct answers, as a result of this activity the student can expect to achieve a better understanding of subtraction, which will guarantee him a better success in future mathematics exercises. As a bonus, he may even acquire some general "debugging" skills which may turn useful the next time he makes a mistake and cannot rely on the teacher for help.

Scenario 3: The tourist's perspective

Suppose that our overall goal is that of learning more about the city we are in, while enjoying ourselves in the process. Though we may be moving around with a destination in mind, we are also willing to relinquish our original goal if something more interesting comes up. This scenario is not so unrealistic as it might appear at first: this is probably what each of us has experienced at some point as a tourist (as long as we were not obsessed with following what our guide told us to see!). Getting lost here may be even welcomed, as an occasion to visit a new part of the city, or as an opportunity to pose a puzzle whose solution may lead us to understand better how the city is laid out, or

even just as an excuse to talk with a local person. Our desired outcome in this scenario is first of all gaining a better understanding of the city -- if we cannot see a certain museum today, we can still do it tomorrow! One may even argue that getting lost in this scenario almost loses its meaning, since the original destination was not so important to start with.

Is there a correspondent of "being a tourist" in the context of education? Though unfortunately it is not easy to find it in a classroom situation, we can find it in the attitude of real scholars and researchers towards knowledge and learning. In their search for knowledge and meaning, specific problems and issues are often perceived as means rather than ultimate goals. Some errors in this context may be perceived as an opportunity to open up new possibilities, to challenge the existing theory, to discover and explore alternatives to the status quo. Philosophers of science such as Kuhn [1970] and Lakatos [1976] have argued that many interesting discoveries in mathematics and science have been generated by the pursuit of what originally appeared as an error - think for example of the creation of non-Euclidean geometries from the failure of original attempts to prove the parallel postulate, or the development of Einstein's theory of relativity in the attempt to explain some inconsistencies of Newtonian physics.

Elsewhere I have argued that this use of errors need not be restricted to geniuses and scientists, and I have provided evidence that it is possible to do so in the context of the high-school mathematics curriculum (ref. Borasi, 1987a, 1987b; Brown-Callahan, 1985). Suppose for example that the students are engaged in creating a definition for

"polygon", a geometric concept they are not yet quite familiar with. Tentative definitions, such as "A geometric figure made of straight lines" or "A closed geometric figure", will likely be proposed by the students and can be discussed by the whole class. Though incorrect with respect to the standard mathematical definition of polygon, these "errors" may provide students with an opportunity to engage in valuable problem solving activities and reflect on the nature of mathematical definitions -- for instance, the tentative definitions suggested may be tested with geometric figures which intuitively represent examples and non-examples of polygons, and somehow improved. Some borderline cases, such as a circle or a "bow-tie" may generate some debate and raise important questions such as: How can we decide whether a figure should be considered an example of polygon before we have even agreed on a definition? How do mathematicians decide whether the definition they have created is appropriate or sufficiently precise? What would be the consequences of agreeing on a different definition of polygon, for example "a polygon is a closed geometric figure made of straight lines?" Even if the lesson may end without producing a "correct" definition of polygon, the students may have learned something even more important about the nature of mathematical definitions and of the way mathematicians operate.

Conditions determining alternative scenarios

The three scenarios identified above should be interpreted not so much as discrete categories but rather as points along a continuum, which seem to occur when we vary the relative importance given to achieving a specific objective versus gaining more general knowledge.

As the examples examined in the previous pages were all legitimate and reasonable, we should not assume that either end of this continuum should be preferable in absolute. For good reasons, none of us would like to be a tourist all the time! On the other hand, our analysis has also shown that some scenarios discourage a constructive use of errors. If we want to allow students to take advantage of the potential of errors to motivate inquiry and generate new learning, we have to make sure that a compatible learning context can be created, at least occasionally, in formal schooling.

An analysis of the conditions that contribute to the creation of specific scenarios thus seems worthwhile at this point. In real life, the presence of real external pressures -- appointments, inflexible working hours, emergencies -- puts us necessarily in a scenario of type 1 most of the time. A similar effect on school learning environments is caused by frequent testing, rigid curricula, streaming on the base of standardized test scores. How many of these pressures are really "external" and inevitable, and which ones could instead be eliminated? A serious answer to this question may be needed, since a "tourist" attitude towards learning cannot be achieved without allowing for a certain amount of leisure.

We also need to realize that the more radical use of errors as a stimulus for exploration and discovery, a characteristic of scenarios 2 and 3, always involves a partial redefinition of the original goal. This will be possible only if the subject is sufficiently in control of the situation to make such a decision. Decisions regarding what to learn, how to do so, and when one will be called responsible for it, are

usually a prerogative of teachers, curriculum developers and administrators. Unless students are invited to participate in these decisions to some extent, our schools will continue to offer only scenarios of type 1.

Even when we recognize the opportunity for learning and inquiry that an error can offer, we may still be unwilling to dedicate the necessary time and effort in pursuing it, unless we perceive the expected outcome as worthwhile. The motivation of a person -- i.e., what are his/her intrinsic interests, needs and values -- will certainly contribute to his/her decision regarding the relative importance of reaching the specific objective originally set versus the pursuit of more general knowledge. In school, the way students' learning is evaluated will inevitably have an influence on this variable. The search for instruments that will allow to measure students' achievements in terms of process and general skills, or in discovery oriented and other creative tasks, is still an open challenge for educators. On the other hand, as tourists and scholars alike seem to be moved by a genuine interest in knowledge, independent from external rewards and recognition, another way to approach this problem is to attempt to increase students' intrinsic motivation regarding academic learning -- perhaps an even greater challenge!

This analysis of the variables playing a role in determining how errors may be perceived was by no means exhaustive. However, it is already sufficient to make educators aware that radical changes in the current organization of schooling may be required if we want students to

change the common interpretation of errors as negative events and learn to benefit from their own errors.

Summary

We are in a position now to step back from the specific analysis of the situations of "getting lost in a city" and "error-making in an educational context", and attempt to organize and summarize our results in more general terms. Both situations can be considered as instances of the more general phenomenon of "going astray while attempting to achieve a specific objective". Our analysis so far has allowed us to uncover the complex relationship between the underlining overall goals which inform our approach to the situation, the interpretation consequently given to the disturbance occurred, the determination of desired outcomes as a result of our actions in response to such disturbance, and finally the conditions associated with each possible scenario.

In Table I, I have attempted to give a schematic representation of these relationships, choosing to identify three specific scenarios, which represent crucial points along a continuum. Each scenario is described along an horizontal line. The characterization of the first scenario, for example, should be read as follows: when the overall goal is that of "reaching a specific objective efficiently", the disturbance will be interpreted as an "obstacle to be eliminated" and consequently the desirable outcome of our actions would be "finding a way to reach the original objective without delay"; the conditions associated with this scenario is "a strong need to reach a specific objective, high time

pressure. a focus on product. no control/flexibility on setting objectives.

TABLE I
Alternative Scenarios

OVERALL GOALS	INTERPRETATION OF EVENT	DESIRED OUTCOME	NECESSARY CONDITIONS
Reach a specific objective efficiently	Obstacle to be eliminated	Finding a way to reach the original objective losing as little time as possible	Strong need to reach a specific objective High time pressure Focus on product No control/flexibility on setting objectives
Reach a specific objective and learn in the process	Obstacle to be eliminated, but from which one can learn for the future	Finding a way to reach the original objective eventually Understanding what went wrong and how the problem could be avoided in the future Gaining new knowledge about the process Gaining new knowledge about the context Learning new skills and heuristics	Need to reach a specific objective No strong time pressure Interest in the process Interest in the context
Learn about a situation; specific objectives are means rather than ends	Springboards for inquiry	Gaining new knowledge about the context Exploring a new area Reaching some new discovery If possible, eventually reach the original objective Learning new skills and heuristics	No real need to reach a specific objective Some leisure Interest in the context Control/flexibility on setting objectives

III.B) Possible Strategies

What do we actually do once we realize that we have made an error, or we are lost? Experiences when we just seemed to "freeze" or act irrationally may be the first ones to come to mind, yet it is also easy to recollect more constructive ways in which we dealt with the situation. Since the outcomes of the experience will clearly depend on the action we finally undertake, it seems important now to examine what alternative strategies are usually available, what outcomes they may be expected to produce, and what variables may affect the subject's decision to engage in each of them. Through the analysis of various real and hypothetical episodes, I was able to identify a few "general strategies" which, I think, represent "reasonable" complementary actions to undertake in response to an error or getting lost. They are: asking someone for help, making use of tools and resources on your own, reflecting on the situation, and engaging in an exploration. Provided that people and tools are available, all these strategies appear accessible in almost any circumstances.

While I was initially expecting to be able to associate a specific outcome to the use of each of these strategies, I soon had to realize that the situation was quite different from my expectations. Rather, one can argue that any of the desirable outcomes identified in the previous section can be pursued by using any of the four strategies, depending on the way the strategy itself is employed. Consequently I realized that my analysis should focus on the variations within the same strategy, rather than on the differences among strategies. In the

following subsections, I will closely examine each of the strategies, focusing on the important variations possible within each one and their relationship with outcomes and goals. The conditions which may affect the evaluation and choice of a specific course of action will also be considered. In this discussion, it will not be possible or even desirable to consider every possible case and combination; the focus would rather be on the close examination of a few significant examples.

A more complete picture of the courses of actions which could be undertaken to achieve a specific outcome will be presented in a schematic and general form at the end of the section (see Table II).

Asking someone for help

Looking for someone which could help us when we are lost may be looked at as an obvious reaction. What is not so obvious, however, is that our goals and perception of the situation may influence what we ask for and how we use the information gathered, and consequently determine the outcomes that "asking for help" may produce. For example, we may just plainly ask for very specific directions to our original destination from where we are now. Or we may try to get additional information about landmarks and possible alternative routes, in case we get confused or lost again along the way. Or we may even engage the person in a conversation about how the city is laid out, ask about the area we are in, get recommendations for a good diner where we could stop for food, and so on. Even once the information is given, our behaviour could differ. We may want to follow the directions provided faithfully as a recipe, or rather just use it as a guideline from which we may occasionally depart whenever desirable -- say, to stop for gas, or to

take on a route we know better, once we hit familiar ground. Obviously the outcomes of our action will vary considerably depending on our decisions.

These considerations may require a reevaluation of the corresponding strategy of "asking the teacher for help", to which many students seem to give first preference when faced with an error (provided of course that they are not given the chance to simply ignore it!). Though as educators we may wish students to be more independent in their academic work, we should not overlook the considerable educational potential of this situation. The error can in fact provide the necessary stimulus and motivation to engage a student in a conversation about some important aspects of the topic under study, taking advantage of the surprise caused by meeting unexpected results (i.e., the error). Discussing an error with a teacher can thus turn into a privileged opportunity to discover some major misunderstanding and realize new connections which may qualitatively improve the student's understanding of the topic. The conversation itself could be conducted along the lines of a Socratic dialogue, and engage the student in creative tasks such as analyzing the potential consequences of the error or even "challenging" it. Suppose, for instance, that the student has produced the incorrect definition of 'circle': "A circle is a closed, continuous, rounded line". The teacher could ask questions such as: What other figures rather than the usual circle fit this description? What is the difference (if there is any) between a 'circle' and a very "rounded" oval? Is there a context in which this

definition of circle could be appropriate? (and indeed, there is such context in mathematics: topology!)

The real problem, however, is that most of the times when the teacher is asked for help regarding an error, the student does not welcome these "digressions" and would tend to disregard them, trying instead to extract from the teacher's response a plain set of instructions which will allow him/her to produce the correct answer. This attitude is not at all surprising if the student is operating in a scenario of type 1, where the overall goal is just to achieve those correct results. It may be worth, however, to examine what other variables may affect how the strategy of "asking someone for help" is applied.

When we get lost, our appraisal of the potential informant, among other things, is likely to affect the way we ask for help and use the response. For instance, we are more inclined to ask a policeman than a kid passing along for directions, and may trust the information provided by the latter only to a certain degree. The likeliness of engaging in idle conversation may even more depend on our informant's willingness to do so. Similarly, we can expect that a student's perception of the teacher's knowledge and interest in spending time on the issue will be an important variable in the educational context. The previous observations also suggest the value of encouraging students to ask for help from people other than the teacher. Working with peers around an error could in fact result in a very educational activity.

A certain degree of familiarity with the situation may also encourage the asking of more appropriate questions, allow for a better

interpretation and assimilation of the information provided, and contribute to establish a "rapport" with the informant -- since the subject will then be able to recognize landmarks, formulate hypotheses that can be tested with the informant, or ask for clarifications at critical points. On the contrary, in a foreign environment the fear of getting confused may motivate a more focussed request for the simplest set of instructions which would allow to reach the original objective.

Personal preferences, past experiences and even cultural background may influence one's decision to look for help, as well as the way this strategy is interpreted. I have noticed for example that people seem more inclined to ask somebody passing by for directions in Italy than in the United States. It is worth to consider the implications of this observation for instruction. Cultural and racial differences, for example, may negatively affect students' perceptions of the teachers as helpful and trustworthy, and consequently decrease their willingness to ask them for help.

Using tools and resources (ex: a map)

When we get lost, for many of us taking out a map and consulting it is almost automatic. This reaction, however, is not shared by everyone, and certainly the correspondent of this strategy in an educational context -- consulting notes, the textbook or references books -- is not too popular among students.

To help understanding this common reluctance against taking advantage of useful tools and resources which may be available, we may start by looking at what is involved in using a map to resolve the problem created by getting lost. First of all, we need to be able to

locate both our current location and our original destination on the map; then we have to identify a feasible route to connect the two points (yet without being able to predict one-way streets or likely traffic jams); this route has to be translated in a series of instructions (turn right, then go straight for 5 blocks...); finally, we have to be able to read or memorize these instructions so that we can follow them while driving. Things get even more complicated if more than one map needs to be consulted and used in order to solve the problem. These steps are covered almost instinctively when you are used to map reading, but may seem formidable for the novice. And this is especially true in their correspondent in an instructional context.

Students may have difficulty locating with efficiency the information relevant to solve their problem, especially when it requires the combination of material contained in different parts of the textbook and or their notes. Translating the information thus found in terms of their specific problem often causes additional difficulty. If we want to encourage students in the use of tools and resources to learn how to deal with their errors more independently, we certainly need to spend more time in school helping them develop strategies to consult written material efficiently on their own. At the same time, however, we can make use of the situation of crisis created by an error to motivate students to learn how to use new tools and strategies, for example by guiding them through the textbook to find the information they need rather than telling them what to do.

Once we are sufficiently familiar with map reading, so that the consultation of a map would not represent a demanding task in itself,

there are still many different ways in which maps can be used when getting lost. Besides just looking for a way to get to our original destination from where we are now, we may be interested in retracing our steps to understand how we got lost, or we may look for patterns in the streets' names and the way the city is laid out which can help us move around with a better understanding and confidence -- with obvious benefits for our future trips. Similarly, students may learn quite different things from their consultation of resources, depending on their definition of the problem created by the error. In any case, since the error presents a contrast with our expectations, it is likely to motivate such consultation and even guide them by generating specific questions.

Exploring

In the appropriate circumstances and if we are in the right state of mind, finding ourselves in an unexpected place because we lost our way may stimulate our curiosity and invite an exploration of our environment. Driving around, even without a precise aim, may result in our discovery of a quaint little park, an interesting used-books store, or an appealing restaurant. Our explorations, however, could also be a little more directed. For example, if the neighbourhood resembles somewhat to a familiar one, we may form some expectations and go around trying to test them out. Or, if we have a general "theory" about how the city is organized, we are likely to apply it to try to find our way around, using all possible cues to make appropriate modifications in our original tentative expectations. Our explorations in these cases are likely to bring along confirmation or refinements of our general

understanding of the city, as well as discoveries about the specific area we happen to be in.

While the rewards of explorations are potentially very high, so is also the risk of considerable frustration -- since we may end up with the only result of finding ourselves even more lost. In order to engage spontaneously in this activity, a person would need considerable confidence in both him/herself and the safety of the environment. All these considerations should be taken in serious account if we believe in the value of students' "intellectual explorations" as an important component of learning, to be introduced more often in school instruction.

The analogy with getting lost also suggests that errors can both motivate and guide students in intellectual explorations. Suppose that a mathematics student has been adding fractions incorrectly by adding numerators and denominators separately -- for example, $2/3 + 1/4 = 3/7$ [Borasi and Michaelsen, 1985]. Could she be encouraged to pursue her idea and create an alternative system of numbers (let's call them "ratios") compatible with this operation? (After all, this is the way we compute batting averages!) Our knowledge of the standard system of fractions can guide this exploration by suggesting questions and working hypotheses : How do we determine whether two ratios are equal? Can ratios be bigger than 1? The experience may in turn motivate the search for the history behind our current number systems, to better understand how mathematicians have engaged in a similar enterprise. As this examples illustrates, "exploring" does not necessarily mean going around blindly, and we can find specific strategies which may help students use

errors to direct their activities (see Borasi, 1987a, for a contribution in this direction within mathematics instruction).

Reflecting

Getting lost necessarily requires us to stop, reflect and reorganize. Something unexpected happened, so we have now to interpret the landmarks and cues around us to understand where we are and what happened -- using all we know, making inferences and testing them. We may also feel the need to revise our original expectations and the "theories" which originated them. In a sense, some form of reflection will necessarily complement whichever of the previous strategies we may employ; however, the direction and extent which this reflection can take, and consequently its potential benefits, may vary considerably.

The most immediate and obvious reaction is probably to question how we could have got lost. We may try in our mind to retrace our steps and reconstruct what happened. As a result, we may be able to get back to the point where we started to go astray, and from there follow the originally planned route. We may also go further and question why we did not reach the original objective as expected: Was it the result of a distraction? Did we inadvertently encounter an exception in the rules usually followed by the street pattern in this city? Were our original directions wrong to start with?

The analogue of this activity in the case of error-making -- i.e., engaging students in the "diagnosis" of their errors -- is now receiving a great deal of attention in some fields of education, such as computer programming and mathematics instruction. Students are even sometimes taught heuristics that can help them "debug" their own errors or conduct

an analysis of their thinking process to discover where they went wrong. These contributions are certainly very valuable for instruction, and may result in students learning important skills as a result of their errors. We should not forget, however, that analysing one's errors has the potential to do more than simply "fixing" the original problem, and should possibly be carried further to question the limitations of the process employed, the possibility of better alternatives, and the consequences for our understanding of the context in which we are operating.

In particular, since errors present a contrast with our original expectations, they may bring along valuable doubt in the appropriateness of the rules and theories we may have operated with so far. This may in turn motivate a revision of those theories and perhaps even suggest some direction for possible modifications -- a role often played by "anomalies" in science, according to Kuhn [1970]. Once we recognize that errors may not always be the result of our faulty behavior, we are also open to the possibility that they may be due to limitations inherent to the context itself, which we have to recognize and accept as necessary -- they may for example be an inevitable exception to the rule, as it happens even in mathematics in cases such as $0/0$!

It is worth noting that the considerable time and leisure required to engage in these reflective activities is rarely there at the moment we make the error, or get lost. Nevertheless, much has to be gained even if the reflection occurs after the fact, when the more immediate needs have been satisfied. In this case, in fact, we may even exploit the additional opportunity of reflecting on our behavior in reaction to

the error, to develop more efficient strategies to deal with error-making in the future.

"Freezing"

To provide a complete picture of what can actually happen when we get lost or make an error, it is important to remind ourselves of less rational and constructive reactions than the ones we have examined thus far. Occasionally, feelings of panic and helplessness could be so overpowering that we seem incapable of doing anything, at least momentarily -- for example, imagine what you could do if you happen to be lost in a foreign city, where maps are not available and people's directions are difficult to understand.

The analogue of this situation, unfortunately, is not so uncommon for students in school. A topic may feel so foreign to a student, that the teacher's pointers may succeed only in creating more frustration, and textbooks or notes appear incomprehensible and thus useless. It is doubtful that in these circumstances the student can do anything constructive with his/her errors, until panic is subsided and the necessary tools and premises are made available. As teachers, it is important to keep this eventuality into consideration. Even if in principle we may believe that errors are something worth analyzing and learning from, and we want students to pay explicit attention to them, we have also to be able to recognize occasions in which it is best if errors are ignored or plainly corrected, so that the student can overcome his/her frustration and start fresh. It is interesting in this regard to report a thesis which some research in second language

acquisition presently supports. These research argues that correcting beginners' errors is at best useless and often counterproductive!

Summary

Our analysis of what can be done in response to going astray in our effort to reach an objective has revealed new aspects of the very complex nature of this phenomenon.

As a result, we have first of all become aware of the influence when discussing goals and desired outcomes of other elements in the context, besides the ones already identified in Section III.A. In particular, a certain degree of familiarity with the situation and the perception of a "safe" environment are likely to encourage a more constructive approach to errors, and may affect our willingness to take advantage of the more radical opportunities for learning that errors may offer.

The previous discussion has only uncovered and elaborated on a few of the possible courses of action that could be undertaken in response to an error/getting lost. In Table II, I have thus reported in a schematic form a more complete prospectus of all these possible alternatives for each of the desirable outcomes identified in Section III.A. Along each horizontal line I have indicated how a specific outcome could be reached by employing a variation of the four "general strategies" for action identified.

TABLE II
Alternative Reactions and Outcomes

GEN. STRATEGIES :				
	: ASKING FOR HELP	: USING TOOLS/RESOURCES	: REFLECTION	: EXPLORATION
OUTCOMES				
Reach the original objective without delay	Ask just for instructions to reach the objective from where you are	Try to find a procedure to reach the objective from where you are	Try to retrace your steps or use cues to to get back on the right track	Try things out blindly hoping to hit something familiar from where to start (LAST RESORT)
Reach the original objective eventually	Ask for directions and other information	Look for procedure(s) to reach objective and pose other questions	Retrace your steps, use cues, to get back to right way and understand what happened	Explore the area you are in, also looking for something familiar to find your way
Understand what went wrong to avoid similar occurrence in the future	Ask questions that may help you understand what went wrong and why	Look for an explanation of what could have gone wrong and why	Retrace your steps, use cues, to understand what happened and why	Explore the area and try to go over your steps to understand what happened and why
Learn new procedures to reach the original objective	Ask for ways to reach the original objective alternative to the one you followed	Look for ways to reach the original objective alternative to the one you followed	After the fact, compare how you finally reached the objective with your original plan	Explore looking for alternative ways to reach the original objective
Develop new skills and heuristics (or motivation to learn those)	Ask for explicit heuristics which could help in this situation (LAST RESORT)	Learn (or plan to learn) how to use new tools which could help you resolve the problem	Reflect on the strategies you employed or could have employed	Engage in explorations which could develop those skills
Know more about the situation/context	Ask information about the context (structure, rules, exceptions, etc.)	Look for patterns, rules, exceptions, and other information on context	Reflect on what happened and what it tells about the context	Explore to better understand the structures, rules, etc. of context
Learn about a new area	Engage in a conversation about the area you unexpectedly encountered	Look for information about the area you unexpectedly encountered	Compare the area you are in with a familiar one, generate hypotheses	Explore the area you unexpectedly encountered
Achieve unexpected discoveries	Engage in an open-ended conversation	Follow up on what did not conform to your expectations	Reflect on what did not conform to your expectations and why	Explore where you are with an open mind

This scheme clearly indicates that all four strategies -- asking for help, using tools or resources, reflecting, and exploring -- are complementary to each other, and could often be used in combination to better exploit the opportunities for learning presented by having gone astray. A closer look at Table II also reveals that many of the actions which would make us benefit from errors/getting lost do not need to be undertaken immediately, and in fact may even be more beneficial if we engage in them after we have resolved the immediate problem of finding a way to reach our original objective.

III.C) Affective Components

An important component of any experience of getting lost is represented by the emotional reactions that accompany it. Not only will those reactions color the experience and make it more or less enjoyable for us, but they also will influence the action we may decide to undertake and consequently its outcomes. Our feelings will in fact play an important role in determining whether the situation is sufficiently safe and worth the risks involved in following the opportunity for learning offered by being lost. In turn, those feelings will be determined by the complex interaction of various elements: our overall goals, time constraints, control over setting objectives, familiarity with the situation, confidence and self-esteem, the outcomes of similar experiences in the past, the availability of tools and helpful people, the audience, and other variables as well. All these considerations point to the realization that in education, too, if we wish students to

benefit in a constructive way from their own errors, we should take into account the importance of the emotions connected with making errors.

On one hand, we should attempt to control to some extent the variables which may affect those reactions. Teachers should try to create conditions in the classroom that could help students at least minimize their first spontaneous negative reactions to errors -- for example, by assuring a sympathetic rather than evaluating audience, and by providing a learning environment where students are encouraged rather than punished for taking risks. Prior experiences with error making in school are usually painful and negative, and students have rarely experienced new discoveries as a result of an analysis of their own errors. To modify these perceptions, we may want students to experience the rewarding results of a constructive analysis of errors at first by choosing an error which the teacher or a scientist may have made (see Borasi, 1987b). This may in fact hopefully reduce feelings of guilt and embarrassment usually associated with making errors in a classroom, in front of your peers. Feelings of helplessness are also likely to be reduced if the students are aware of alternative strategies which could allow to respond constructively to errors (such as the ones we have discussed in section IIIB), and if they perceive those strategies as actually available to them -- this may require, for example, the presence of knowledgeable and trustworthy people and the availability of tools they feel confident to use. Rather than ignoring or denying the negative emotions likely to be experienced when an error is first discovered, we should also try to make students explicitly aware of these feelings and discuss ways in which they can be overcome.

IV - Conclusions

Viewing error-making as analogous to getting lost in a city has brought us to broadly revise our whole approach to the problem of student errors in an educational context. The previous analysis has in fact revealed that errors can present the opportunity for valuable learning activities, provided that they are used constructively and certain conditions are verified. Rather than looking for strategies which would help teachers understand the causes of student errors and eliminate them, our attention has thus focussed on "how can we provide a learning environment in which students can effectively take advantage of their own errors".

Let us briefly summarize the major results reached through the explicit development of this new metaphor for error-making. First of all, we have been able to recognize a number of valuable outcomes which can result from a constructive use of errors in instruction:

- . reaching (eventually) the original objective we had set to achieve;
- . understanding what went wrong, so that we are less likely to repeat the same mistake in the future;
- . identifying new alternative procedures which would allow to reach the original objective;
- . developing general problem solving and thinking skills and heuristics;
- . gaining knowledge about the context we are operating in;
- . learning about a new topic or area;
- . achieving some unexpected discoveries.

Which of these outcomes we may perceive as desirable will essentially depend on our overall goals in the situation, and in particular on the relative importance given to the immediate achievement of the specific objective we originally set to achieve versus gaining more general knowledge and skills. This balance will in turn influence in what measure we will perceive the error as "an obstacle to be eliminated" and/or "a springboard for inquiry and learning".

Some constructive strategies that can be employed in response to an error have also been identified; they are:

- . asking someone for help;
- . using appropriate tools and resources available;
- . engaging in explorations;
- . reflecting on the situation, bringing to bear what we already know.

These four strategies have all the potential for contributing to the achievement of each of the desirable outcomes listed above. As shown in Table II, however, the exploitation of this potential will depend essentially on the kind of questions we pose and set to answer using the chosen approach -- in other words, on the way in which we define the problem created by having made the error.

It is disturbing to realize that currently in school we do not provide students with the variety of scenarios which could help them appreciate the creative potential of errors and develop appropriate strategies to deal with errors constructively in different circumstances. Students' perception of the overall goal of schooling is too often limited by a focus on producing specific "products", rather than on the attainment of general skills, knowledge and understanding.

It is not surprising that in this context errors are perceived by both students and teachers as a nuisance at best, feelings of panic and shame are often connected with error-making, and asking the teacher to tell the correct answer and reexplain how to "get there" is perceived as the most efficient strategy for dealing with errors. There is little hope in this scenario to make the students appreciate that an analysis of their errors can be valuable.

If we wish to change this situation, and create opportunities in school for students to recognize, appreciate and use the educational potential of errors, we need to provide more appropriate conditions. The analogy with getting lost has been helpful in this direction, too, by suggesting a number of variables that may affect students' perceptions of the goals of schooling and of the value of engaging in the inquiry stimulated by errors. The following conditions seem especially important to encourage students to view errors as springboards for inquiry and learning:

- . lower priority set on the attainment of specific objectives;
- . some control and flexibility from the part of teachers and students on modifying set objectives;
- . motivation and interest in learning about the process;
- . motivation and interest in learning about the context;
- . some leisure in learning;
- . some familiarity with the situations in which the error occur;
- . perceived safety of the environment in which the experience occurs;
- . previous positive experiences as a result of errors;
- . a supportive audience;

- . helpful people available for help;
- . useful tools available.

It may important to remark that a change in the directions indicated may indeed present a radical challenge to the way learning and instruction is structured in the current system of schooling.

To conclude, we would like to point out that the metaphor of "error-making as getting lost" provides not only new perspectives to examine the potential roles of errors in instruction, but also a powerful means to communicate effectively this new awareness to other teachers and educators. The teachers themselves may in fact be encouraged to engage in the reflection of what happens when one gets lost and in the development of their own implications of this metaphor for dealing with errors in an educational context. Relating to their own personal experiences of getting lost may put teachers in a better position to relate with their students' feelings, conceptions and reactions when making errors in school. It can also open new vistas about how errors can be used in instruction, and what conditions may be necessary to do so. This in turn may result in real and long-lasting changes in the way errors are treated in formal instruction.

My experiences so far, conducted both in the context of informal communications and teacher preparation courses, confirm this belief and encourage me to employ the generative use of educational metaphors as a strategy for teacher education.

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